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Calibration of VISSIM Roundabout Model: A Critical Gap and Follow-up Headway Approach

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By

Zhixia Li, Ph.D.
Research Associate
1249A Engineering Hall, 1415 Engineering Drive, Madison WI 53706
Tel: (513)484-2991; Fax: (608)262-5199
Email: zli262@wisc.edu

Michael DeAmico, M.S.
Traffic Engineer
AECOM, 1350 Deming Way, Suite 100, Middleton WI 53562
Tel: (608)828-8147; Fax: (608) 836-9767
E-Mail: michael.deamico@aecom.com

Madhav V. Chitturi, Ph.D.
Assistant Researcher
B243 Engineering Hall, 1415 Engineering Drive, Madison, WI 53706
Phone: (608)890-2439, Fax: (608)262-5199
Email: mchitturi@wisc.edu

Andrea R. Bill
Associate Researcher
B243 Engineering Hall, 1415 Engineering Drive, Madison, WI 53706
Phone: (608)890-3425, Fax: (608)262-5199
Email: bill@wisc.edu

AND

David A. Noyce, Ph.D. PE
Professor
1204 Engineering Hall, 1415 Engineering Drive, Madison, WI53706
Phone: (608)265-1882, Fax: (608)262-5199
Email: noyce@engr.wisc.edu

Traffic Operations and Safety (TOPS) Laboratory
Department of Civil and Environment Engineering
University of Wisconsin-Madison

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1 **ABSTRACT**

2 VISSIM roundabout models have been widely applied in practice to facilitate analyzing the
3 operational performance of roundabouts. To prepare a VISSIM roundabout model for analysis,
4 an essential prerequisite is to calibrate the model by adjusting parameters until real-world
5 roundabout operations are reproduced in the simulation model. Previous calibration research has
6 used qualitative analysis to study the impact of VISSIM parameters on roundabout capacity.
7 Comprehensive calibration guidelines, parameter values based on field data, and quantitative
8 sensitivity analyses of parameters are necessary to facilitate accurate modeling of roundabouts.
9 This paper addresses these important needs. Speed trajectories of free-flow entering vehicles
10 were collected in the field using a radar sensor. Analysis identified that the approach to a
11 roundabout entrance can be divided into four speed zones reflecting different stages of drivers'
12 deceleration maneuver. Location, length, speed distribution, and deceleration rate parameters for
13 the VISSIM Reduced Speed Areas (RSA) were determined through the analysis of the radar
14 data. Comparisons between Conflict Areas (CA) and Priority Rules (PR) were also investigated,
15 and revealed that using PR can result in more consistent and repeatable gap acceptance behavior.
16 In addition, the impact of VISSIM parameters on critical gap and follow-up headway was
17 quantitatively analyzed through sensitivity analysis of minimum gap for PR, speed distribution
18 and deceleration rate for RSA, and additive and multiplicative settings for the Wiedemann 74
19 model. Numerical recommendations for calibrating VISSIM roundabout models were ultimately
20 developed, and validated via a case study.

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42 INTRODUCTION

43 In recent years, as many intersections in the U.S. have been converted to, or originally built as
44 roundabouts, analyzing roundabout operations and safety has drawn extensive attention from
45 practitioners and researchers. Most commercial microscopic traffic simulation software packages
46 offer the capability of building roundabout simulation models. VISSIM roundabout models have
47 been heavily discussed as one of the most widely applied microscopic simulation packages for
48 modeling roundabouts (1-12). In order for a simulation model to provide useful output, an
49 essential prerequisite is to prove that the established simulation model can accurately mimic real-
50 world traffic operations. In other words, the simulation model has to be calibrated through
51 adjusting model parameters and be validated through comparison with field ground truth data
52 before the model can be used for analysis.

53 Previous studies have summarized that the settings of three elements in VISSIM have
54 critical impact on the operational performance of roundabout simulation models (3-5, 9-12).
55 These elements include: (1) Priority Rules (PR) or Conflict Areas (CA), which control the
56 yielding logic; (2) Reduced Speed Areas (RSA), which provide temporary speed control over a
57 short roadway distance; and, (3) Wiedemann 74 and 99 car following models, which can fine-
58 tune the simulated car-following behavior.

59 Much research has been conducted to explore methods for calibrating VISSIM
60 roundabout models. The primary method of calibration was repeatedly adjusting parameter
61 settings of VISSIM elements until a calibration goal was reached (3-5, 9-12). The typical
62 calibration goal used in previous studies was to match the capacity curve obtained from the
63 simulation model to the field-observed capacity curve (3-5), while some other studies matched
64 simulated travel time or speed to field data (10-12). Most of these studies focused on different
65 VISSIM settings for calibration. Only a few studies investigated using field collected data as
66 input into the roundabout simulation model with regard to calibration, and discussed the model
67 validation using field data. In summary, previous research has not adequately fulfilled the need
68 for comprehensive calibration guidelines, recommended simulation parameter values based on
69 field data, or in-depth sensitivity analyses that quantify the impact of changing various VISSIM
70 parameter settings on simulated capacity.

71 In this context, the objective of the paper is two-fold: (1) quantitatively investigate the
72 sensitivity of change of roundabout capacity under different settings of VISSIM elements; and,
73 (2) provide quantitative guidance on selection of VISSIM elements during the calibration
74 process, and recommend field-estimated parameter values for calibration.

75

76 LITERATURE REVIEW

77 The earliest documented study on calibration of VISSIM roundabout models was a paper back in
78 2003, in which Trueblood and Dale gave a good overview of the basics of how VISSIM works
79 with regards to modeling single and multilane roundabouts (2). However, most information
80 provided was qualitative. Particularly, the validation of PR settings was based on trial and error
81 only by viewing the animation file produced by VISSIM (2). Instead of PR, Schroeder
82 investigated CA in his study, and described a methodology for calibrating the roundabout
83 simulation model through sensitivity analysis of CA's gap parameters (3). Schroeder's
84 sensitivity analysis gives qualitative results of the impact on the intercept and slope of the
85 capacity curve when changing the inputs of various VISSIM parameters. Schroeder's analyses
86 were based on one simulation run per experiment with varied volume inputs during different

87 periods of the experiment. Although this single run approach can save calibration time (3), fewer
88 observations were collected than in other studies (4), and circulating flows above 1400 veh/hr
89 were not obtained.

90 Cicu et al. used PR to model a two-lane roundabout in VISSIM (5). No sensitivity
91 analysis was conducted. The major contribution of this study is that researchers tried to find
92 proper parameter estimates using field-collected data, including critical gap and speed. However,
93 only the simulation output capacity was compared with the capacity curve recommended by
94 National Cooperative Highway Research Program (NCHRP) Report 572 (6). Validation may
95 have been stronger if they had compared the capacity output from the calibrated VISSIM model
96 to field-observed capacity.

97 Wei et al. experimented with both PR and CA in VISSIM (4) and concluded that both can
98 be applied in VISSIM roundabout models. However, they mentioned that CA can occasionally
99 produce situations where a circulating vehicle yields to an entering vehicle (4). Multiple runs
100 with different random seeds were used (4). Wei et al. analyzed the impact of VISSIM parameters
101 on critical gap, follow-up headway, and eventually the capacity curve, although the method of
102 estimating critical gap and follow-up headway was not discussed in detail. The results were
103 mostly limited to the qualitative level by only describing whether an increasing or decreasing
104 trend was observed. Also, the selection of parameter values was based on the default values
105 recommended by VISSIM rather than from field observations. Overall, the study is one of the
106 most comprehensive studies with regard to VISSIM model calibration in the literature. Wei et al.
107 finally suggested that in future research more detailed investigation into critical gap and follow-
108 up headway should be conducted (4).

109 From a methodology perspective, Duong et al. developed a general framework for any
110 simulation model calibration, including VISSIM (7). Duong et al. used time to collision (TTC),
111 rather than the capacity, as the performance measure to calibrate a roundabout simulation model,
112 which made the methodology more appropriate for model calibration from a safety perspective.

113 In addition to the aforementioned research efforts, which were dedicated to VISSIM
114 roundabout model calibration, other studies were conducted using VISSIM roundabout
115 simulation models. Bared and Afshar investigated roundabout capacity using a multilane
116 roundabout model in VISSIM (8). No calibration of the simulation model was mentioned in the
117 paper. Fortuijn investigated capacity for turbo roundabouts using VISSIM (9). Calibration was
118 achieved by modifying the Wiedemann car-following model and PR to achieve a fit between the
119 distributions of accepted gaps, rejected gaps, follow-up headways, and headways in circulating
120 traffic. Valdez et al. investigated roundabout delay with unbalanced approach volumes using a
121 two lane roundabout from the NCHRP Report 572 dataset coded in VISSIM (10). Calibration
122 was performed by adjusting the CA gap times until the travel time distribution in VISSIM
123 matched the travel time distribution of the field data. Gallelli and Vaiana also conducted research
124 on delay with VISSIM by evaluating the effects of roundabout geometry on delay (11). In
125 another study conducted by them, the simulation model was calibrated by using speed as the
126 performance measure (12). Gagnon et al. investigated the calibration abilities of different
127 software packages, including PARAMICS and VISSIM (13). Al-Ghandour et al. used the
128 VISSIM roundabout model to develop conflict models to predict crashes at single-lane
129 roundabouts (14). Lu et al. studied the impact of the pedestrian crosswalk on the capacity of a
130 roundabout using VISSIM simulation (15).

131 In summary, although considerable research has been conducted on VISSIM roundabout
132 models, there is still a lack of a comprehensive calibration guideline with quantitative
133 recommendations on the selection of different parameter values in term of calibrating VISSIM
134 roundabout models.

135

136 **METHODOLOGY**

137 **Data Collection**

138 Field data collection is of great importance to provide reasonable field estimates for VISSIM
139 parameters as well as observed ground truth capacity data. The chosen study site was a congested
140 two-lane roundabout in De Pere, Wisconsin. As shown in the top portion of Table 1, the De Pere
141 roundabout is located at the intersection of State Trunk Highway 32 and 57.

142 Video cameras were set up in field to capture vehicle events including arrival, entry, and
143 exit, as well as conflicts between entering and circulating vehicles of the NB and EB approaches.
144 Based on the recorded time stamps of these events, one-minute circulating, and entering flow, as
145 well as critical gap and follow-up headways were then derived. Specifically, the estimation of
146 critical gaps was based on the maximum likelihood (ML) method (16-18), assuming that the
147 critical gap follows a log-normal distribution. As a result, the top portion of Table 1 summarizes
148 the observed critical gaps and follow-up headways for passenger car and heavy vehicles for the
149 NB approach, which is the major study approach of this research due to its high congestion level.

150 The middle portion of Table 1 summarizes the capacity data collected at the left lanes of
151 the NB and EB approaches of the De Pere roundabout. All capacity data are 1-minute-based, and
152 were collected under queuing conditions and then converted to passenger car equivalent using
153 the conversion factor of one heavy vehicle equivalent to two passenger cars. Due to the traffic
154 pattern at the roundabout during the data collection period, no circulating flows below 400 pc/hr
155 were observed at the NB left lane, while no circulating flows above 1000 pc/hr were observed at
156 the EB left lane. Since the validation of a roundabout simulation model requires the ground truth
157 capacity data to have a full range of circulating flows, all capacity data from EB (low circulating
158 flows) were merged into the NB capacity data in order to prepare a complete dataset of ground
159 truth capacity, as shown in the right middle portion of Table 1. The merge is based on the fact
160 that the EB left lane has similar critical gap and follow-up headway with the NB left lane.

161 In addition to the capacity and gap acceptance data, free-flow speed data were collected
162 at a roundabout approach in Oshkosh, WI using a microwave radar sensor. The purpose of
163 collecting the speed data is to provide field estimation for input parameters of the RSAs in
164 VISSIM. This roundabout approach was selected for speed data collection because it has similar
165 entrance 85th percentile speed with that of the De Pere roundabout. Also, there is no horizontal
166 curve on the approach to the roundabout (upstream of the roundabout). Hence, the geometric
167 effect on the speed is minimized. The radar sensor scanned the approaching traffic every 0.3 sec
168 covering distances up to 500 feet from the sensor. The corresponding location and speed data
169 were recorded. Note that all data pertaining to the non-free-flow vehicles, which are vehicles that
170 stopped during the entire course of approaching and entering the roundabout, were dropped
171 during the data reduction process. The bottom portion of Table 1 shows the observed speed
172 profiles of free-flow entering vehicles. The mean and 85th percentile entry and upstream speeds
173 of a total of 539 observed free-flow vehicles are summarized in the bottom portion of Table 1.

174 **TABLE 1 Summary of Field Collected Data**

Top Portion	Acceptance Data	Passenger Car	NB							
			Left Lane				Right Lane			
			n*	t _c * (s)	n	t _f * (s)	n	t _c (s)	n	t _f (s)
		Truck	648	4.3 (1.0)	638	3.1 (1.2)	428	3.6 (0.9)	406	3.0 (1.2)
			58	5.2 (1.2)	36	3.7 (1.2)	15	4.8 (2.0)	15	3.4 (0.8)

* n denotes the sample size; t_c denotes critical gap; t_f denotes follow-up headway; ()denotes standard deviation.

Middle Portion	Capacity Data	Critical Lane (Left Lane)	NB	EB	Combined

Bottom Portion	Speed Data	Free-Flow Entering Vehicles (0.3 s-based data)	Speed vs. Distance to Radar Sensor	Statistics of Free-flow Entering Vehicles	
				Sample Size (veh)	539
		Mean Upstream Free-flow Speed (mph)	28.7		
		85 th Percentile Upstream Free-flow Speed (mph)	32.0		
		Mean Entry Free-flow Speed (mph)	12.1		
		85 th Percentile Entry Free-flow Speed (mph)	16.0		

175 **Critical Gap and Follow-up Headway Based Sensitivity Analysis**

176 Most previous studies used capacity curve based sensitivity analysis when exploring the
 177 calibration guidelines for VISSIM roundabout model. Typically, the output capacity cloud from
 178 VISSIM (i.e., dots representing entering flow versus circulating flow) or the cloud's regression
 179 curve was used as the only performance measure in the sensitivity analysis. Due to the fact that
 180 the capacity cloud is a distribution, most results of these sensitivity analyses were hard to
 181 quantify other than qualitatively describing the change of the capacity curve's intercept and
 182 slope.

183 According to the Highway Capacity Manual 2010, the capacity of the critical lane of a
 184 multilane roundabout is an exponential function of critical gap (t_c) and follow-up headway (t_f), as
 185 expressed by the following equation (19).

$$186 \quad C_{pce} = Ae^{(-Bv_c)} \quad (1)$$

$$A = \frac{3600}{t_f}$$

$$B = \frac{t_c - (t_f / 2)}{3600}$$

187 Where, C_{pce} = lane capacity, passenger car equivalent (pc/hr)
 188 v_c = conflicting flow (pc/hr),
 189 t_c = critical gap (s), and,
 190 t_f = follow-up headway (s) .

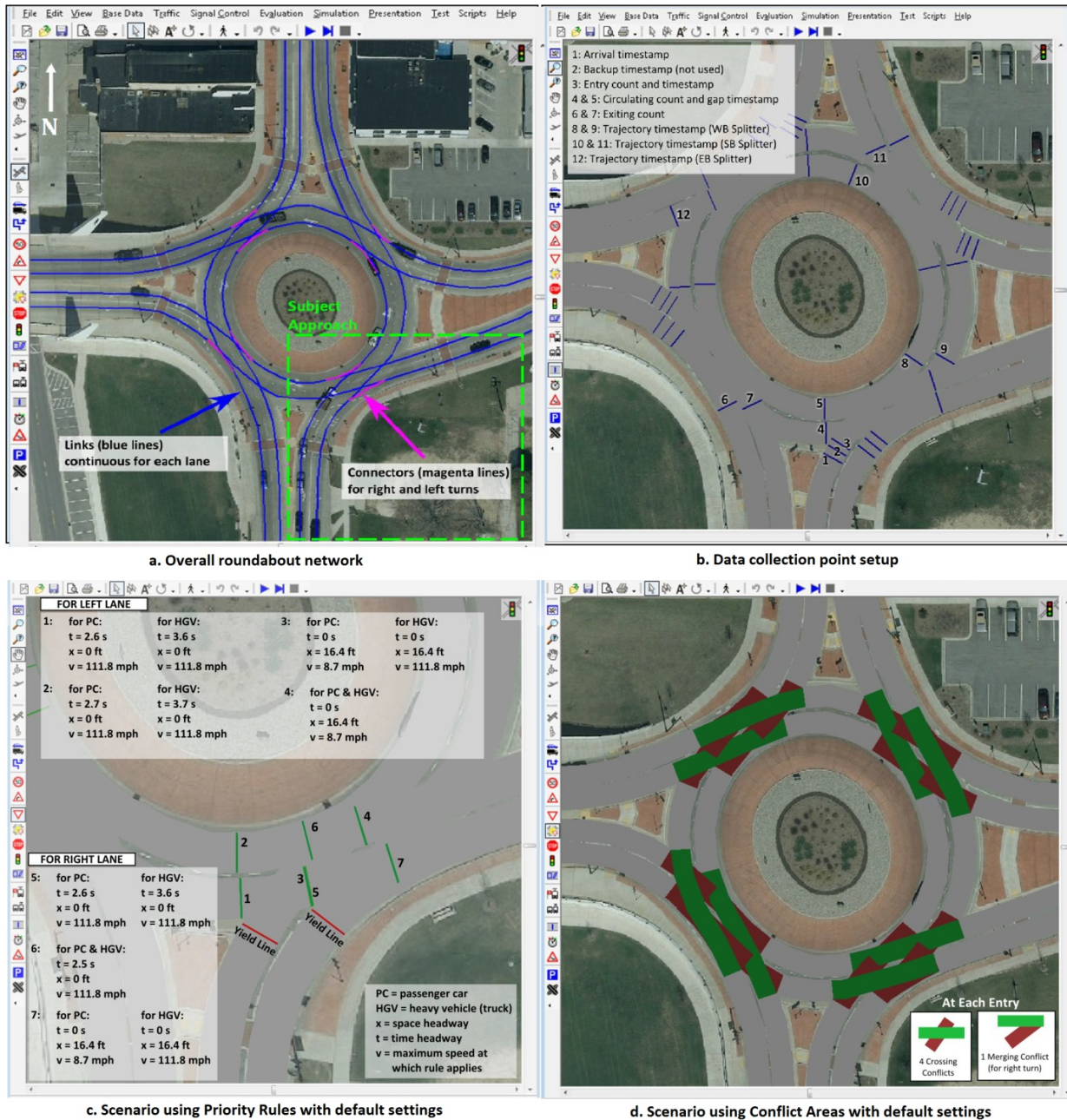
191 Since t_c and t_f are the only parameters of the capacity model, the roundabout capacity can
 192 hence be simply determined by these two parameters. Considering that t_c and t_f are much easier
 193 to describe quantitatively than the capacity cloud, they are a better quantitative performance
 194 indicator in sensitivity analysis, replacing the traditional capacity cloud. Based on this fact, in
 195 this research, t_c and t_f were estimated from VISSIM's output data using similar methods as used
 196 in field data collection. They were used as the major performance measure in the sensitivity
 197 analysis and the calibration process.

199 **Setup of the Roundabout Model in VISSIM**

200 As shown in Figure 1.a, the De Pere roundabout model was coded in VISSIM based on its aerial
 201 map. The desired speeds for all links were set to a distribution ranging from 22 to 36 mph, with
 202 26 mph and 29 mph as the 50th and 85th percentile speeds, which matched expectations for a
 203 25 mph speed limit. Figure 1.b illustrates the locations of the data collection points that were
 204 placed to facilitate collection of traffic flow data as well as timestamps of vehicles' gap
 205 acceptance events. The timestamps were used to estimate the critical gap and follow-up
 206 headways.

207 In order to investigate the performance difference between PR and CA, two default
 208 network scenarios using PRs and CAs were created. One used PRs and the other used CAs to
 209 define the yielding logic. All other VISSIM parameters were identical. In all cases, the NB
 210 approach was used for the study approach, and data from the left lane was specifically chosen for
 211 sensitivity analysis. Figure 1.c shows the layout of the PRs defined for the NB study approach,
 212 using the priority rule settings recommended in the VISSIM User's Manual (20). One exception
 213 is that the setting of the minimal gap for the right lane was changed to 2.5 sec from the
 214 recommended 1.8 sec to obtain more realistic yielding behavior. Note that all these default

215 settings were temporary, and were changed in the sensitivity analyses, which are to be discussed
 216 in detail in the following section.
 217



218
 219

FIGURE 1 VISSIM roundabout model setup.

220 All simulation experiments performed in this research were based on simulation runs of
 221 1800 sec (30 minutes) at a resolution of 10 time steps per simulation second. A five minute
 222 warm-up time was included in each run to allow traffic to stabilize before collecting data
 223 between 300 sec and 1,800 sec (25 minutes). Each run was used to obtain the entering flow
 224 under one regime of circulating flow. A total of fifteen flow regimes were used to generate data

225 throughout a range of practical circulating flows, with 10 simulation runs using different random
226 seeds per regime, resulting in a total of 150 simulation runs per experimental trial. The first two
227 flow regimes correspond to circulating flows of 25 veh/hr/ln and 100 veh/hr/ln, respectively. For
228 each subsequent regime, 100 veh/hr/ln were added starting from flow regime #3. Flow regime
229 #15 has a circulating flow of 1400 veh/hr/ln.

230 Because the NB approach was selected as the study approach, the entry volume demand
231 of NB was fixed at 2500 veh/hr in each lane to ensure that there was always sufficient entering
232 demand at this approach. The circulating flow was only from the EB approach (adjacent entry) to
233 allow an easy control of circulating flow rate. Considering that VISSIM provides options to
234 model the operations of cars and heavy vehicles separately, such as establishing separate PRs
235 and reduced speed areas, the critical gaps and follow-up headways for cars and heavy vehicles
236 can therefore be calibrated separately. Based on this consideration, all simulation runs in this
237 research used vehicle composition of 100% passenger cars in order to simplify the process of
238 exploring the calibration approach. The difference between cars and heavy vehicles is that the
239 heavy vehicles have larger critical gaps and follow-up headways. Therefore, the calibration
240 recommendations developed based on cars can be simply used for calibrating heavy vehicles by
241 setting the calibration goals of longer critical gaps and follow-up headways.

242

243 **Method for Estimating Critical Gap and Follow-up Headway in VISSIM**

244 Gaps were calculated as the time difference between timestamps of vehicles crossing data
245 collection points 4 and 5 as shown in Figure 1.b. Locations of points 4 and 5 matched the PR
246 conflict markers for the left lane. Gaps were then indexed chronologically. Finally,
247 characteristics of the gaps accepted and rejected for each vehicle were computed in order to find
248 maximum likelihood method estimates of critical gap. Headway between two entering vehicles
249 was considered as a follow-up headway if the two vehicles accepted the same gap. The
250 differences between timestamps of vehicles crossing data collection point 3 (See Figure 1.b)
251 during a single gap were used to estimate average follow-up headway. In summary, the same
252 method for estimating the field-observed critical gap and follow-up headway was used for
253 VISSIM data.

254

255 **ANALYSIS AND RESULTS**

256 **Analysis of Driver's Speed Reduction Behavior**

257 While the placement of RSAs has been investigated previously, most suggested placing RSAs on
258 the entrances of roundabouts (2-4, 12). However, how far from the yield line the RSA should be
259 placed, and what speed distribution and deceleration rate should be used was not specified; rather
260 they were based on experience. This research tried to give recommendations on the placement
261 and settings of RSAs based on speed trajectory data collected in field.

262 Figure 2.a shows the speed profiles of 65 free-flow entering vehicles as they approached
263 the roundabout. Different colors are used to represent different vehicles. These 65 vehicles were
264 randomly selected from a total of 539 observed free-flow vehicles to achieve an easier
265 recognition of speed patterns. However, in the detailed data analysis, the full sample of 539
266 vehicles was used. In Figure 2.a, a relatively level speed profile was observed when the distance
267 from yield line was greater than 160 feet, which indicated that constant speeds were maintained
268 by vehicles. A distinct change in slope in the speed profile began at around 160 feet from the
269 yield line and continued smoothly, which indicated a continuous and consistent deceleration

270 maneuver by drivers. The slope became less steep when the distance reached about 25 feet from
 271 the yield line. This indicated a trend that drivers started to end the deceleration and tried to
 272 maintain a constant speed. When the distance reached around 8 feet from the yield line, speed of
 273 nearly half of the vehicles started to increase, indicating that vehicles began to accelerate to enter
 274 the roundabout. Based on these patterns, the entire roundabout approach could be approximately
 275 divided into four speed zones, namely:

- 276 • Constant Speed Zone (>160 feet from yield line),
- 277 • Deceleration Zone (25-160 feet from yield line),
- 278 • Reduced Speed Zone (8-25feet from yield line), and
- 279 • Speed Up Zone (0-8 feet from yield line).

280 Assuming that the deceleration rate within the Deceleration Zone is fixed (similar assumption
 281 used in VISSIM RSAs), each vehicle's deceleration rate can be computed using the following
 282 equation:

$$283 \quad a = \frac{v_e - v_0}{t_e - t_0} \quad (2)$$

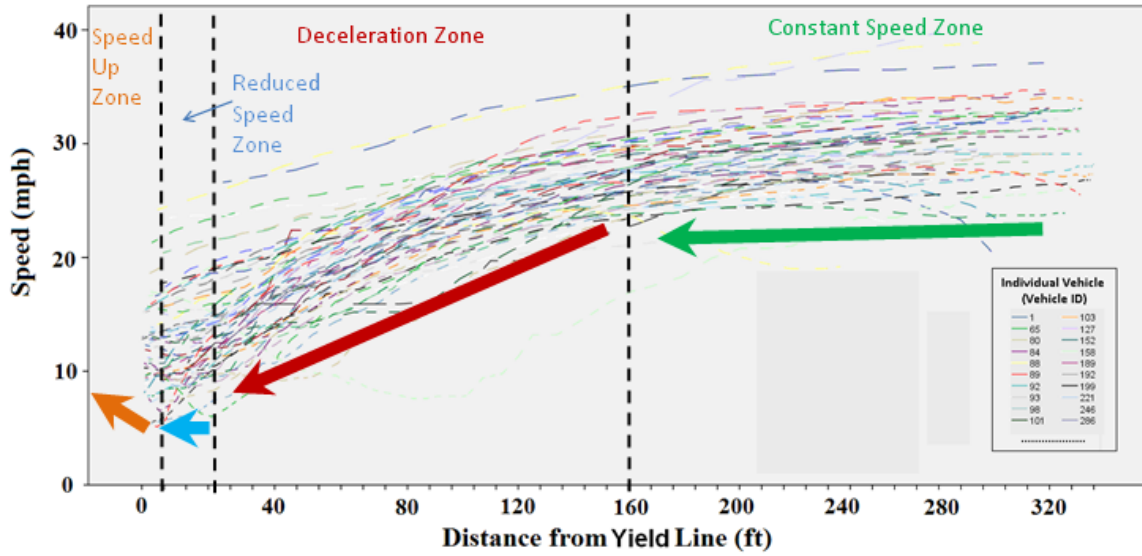
284 Where, a = deceleration rate (ft/s²);
 285 v_e = exiting speed (ft/s);
 286 v_0 = entering speed (ft/s);
 287 t_e = exiting timestamp (t); and,
 288 t_0 = entering timestamp (t);

289 Figure 2.b shows the distribution of deceleration rate in the Deceleration Zone for all
 290 observed free-flow vehicles. The mean deceleration rate and standard deviation were found to be
 291 4.19 ft/s² and 1.48 ft/s², respectively.

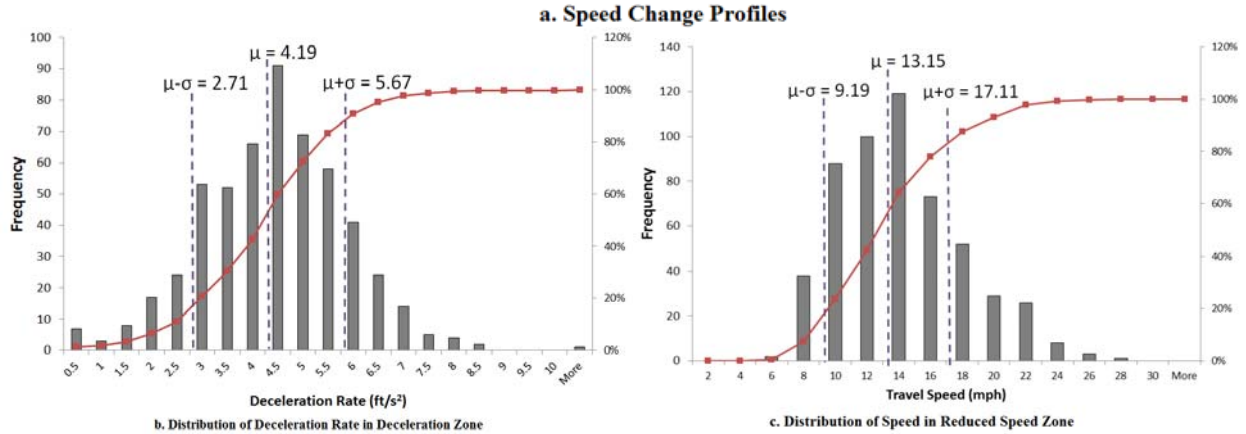
292 According to Figure 2.a, vehicle speeds in the Reduced Speed Zone vary slightly at
 293 different distances; however they are relatively stable when compared to the speeds in the
 294 Deceleration Zone. Therefore, the assumption was made that each vehicle maintained near
 295 constant speed in the Reduced Speed Zone (similar assumption used by VISSIM RSA), and the
 296 constant speed (termed as travel speed) for each vehicle could be computed by taking the
 297 average of each vehicle's speed measurements within the Reduced Speed Zone. Figure 2.c shows
 298 the distribution of travel speed in the Reduced Speed Zone. The mean travel speed and standard
 299 deviation were found to be 13.15 mph and 3.97 mph.

300 According to the definition of RSA in VISSIM user's manual (20), the location of the
 301 entrance RSA should exactly overlap with the Reduced Speed Zone as shown in Figure 2.a.
 302 Deceleration rate and speed distribution parameters therefore correspond to the observed
 303 deceleration rate in the Deceleration Zone and the travel speed distribution in the Reduced Speed
 304 Zone, respectively. Figure 2.d illustrates the layout of the entrance RSA in the VISSIM
 305 roundabout model. Based on the findings from Figure 2.a, the length of the RSA equals the
 306 length of the Reduced Speed Zone, i.e., 17 feet. The end boundary of the RSA is located at 8 feet
 307 from the yield line. The speed distribution in the RSA conforms to the cumulative speed curve as
 308 shown by the red curve in Figure 2.c. The default deceleration rate has been changed to 4.19 ft/s²
 309 as it is the mean deceleration rate observed in field.

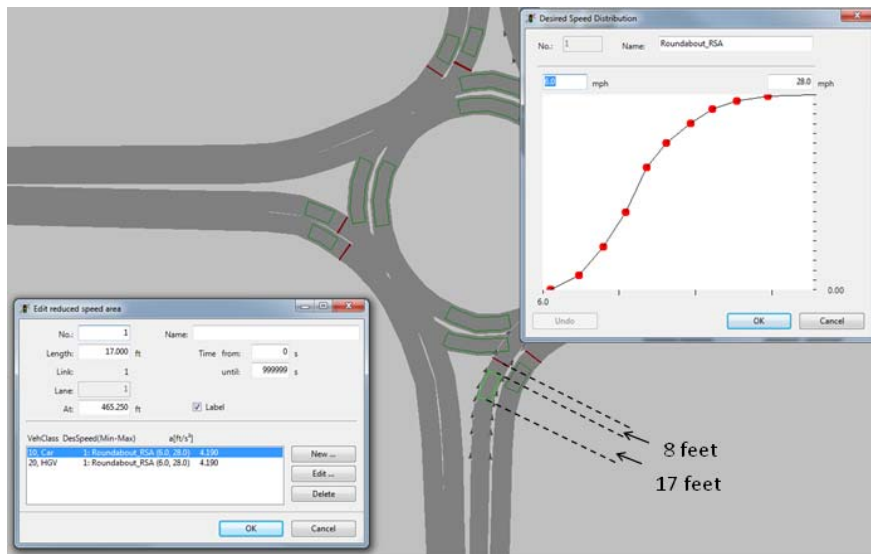
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d. placement of RSA

FIGURE 2 Parameters for VISSIM reduced speed areas.

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317 In addition to the placement of the entrance RSA, recommendations by Trueblood and
318 Dale were also taken into account in this research in regards to placement RSAs within the
319 circulatory roadway (2). Instead of having large, continuous RSAs, smaller RSAs at the splitter
320 islands were defined to enable vehicles to realistically travel at speeds typically observed within
321 roundabouts (2). The placement of these circulatory RSAs is illustrated in Figure 2.d.

322

323 **Analysis of Performance of Conflict Areas and Priority Rules**

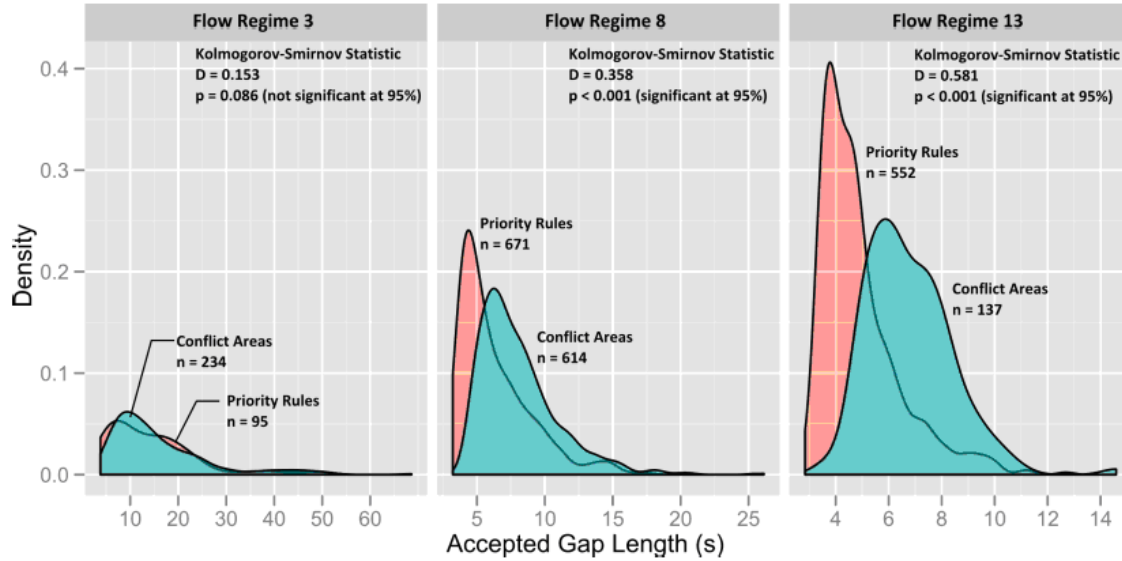
324 VISSIM provides two options for modeling roundabout's right-of-way, namely PR and CA.
325 VISSIM user's manual gives examples of using both PR and CA for modeling roundabouts. This
326 section aims at providing assistance to the practitioners and researchers on which to choose
327 between these two alternatives through analyzing the difference in yielding behavior resulting
328 from the use of PR and CA.

329 Wei et al. found that the use of CA may cause the situation where a circulating vehicle
330 yields to an entering vehicle (4). In this research, it is revealed that the occurrence of the yielding
331 behavior of circulating traffic under CAs can be eliminated through careful placement of the
332 links and connectors in the outer lane. Specifically, the placement of the right turn connector was
333 the key. If the right turn connector was drawn such that it started before the yield line, so that
334 only the front part of the first vehicle waiting in queue would be on the right turn connector, the
335 yielding phenomenon of the circulating traffic was alleviated.

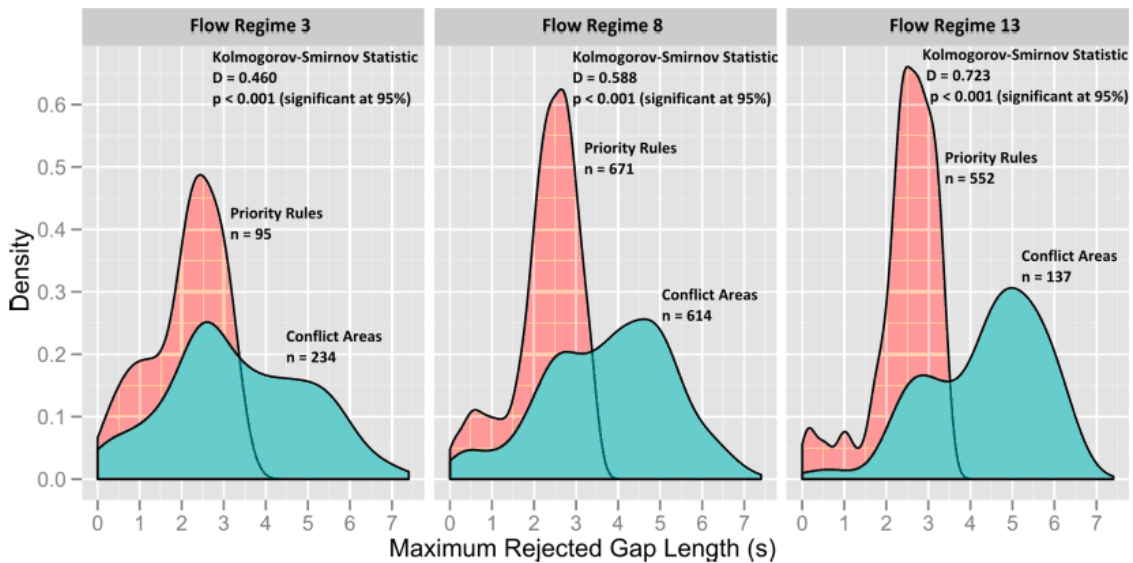
336 Considering that the yielding issue has been fixed for the CAs, a further comparison of
337 the gap acceptance distributions between CA and PR was performed to analyze their
338 performance from the perspective of consistency in driver's gap acceptance behavior. Figures 3.a
339 and 3.b show the distributions of the accepted gaps and the largest rejected gaps under PR and
340 CA scenarios. Note that the CA scenarios were adjusted to eliminate the yielding phenomenon of
341 the circulating traffic). Distributions were compared by the two-sided Kolmogorov-Smirnov (K-
342 S) statistic to test for statistically significant differences between the distributions, measured by
343 the distance, D, statistic. For accepted gap distribution, low circulating flows were similar
344 between PR and CA. However, differences in accepted gap distribution increased with the
345 increasing circulating flow, which is indicated by the increasing D statistic. For the largest
346 rejected gap distribution, significant differences were observed at all circulating flows. PR can
347 also be seen to have narrower distributions, indicating more uniform behavior in accepting and
348 rejecting gaps. CA on the other hand, showed a wider distribution, even rejecting large gaps of 6
349 sec or more regardless of conflicting flow regime. All these facts indicate that driver's gap
350 acceptance behavior under PRs is more consistent and repeatable than under CAs. The
351 consistency and repeatability under PRs concurs with the field-observed normally distributed
352 accepted and rejected gaps which are reported in the previous studies (17, 18, 21).

353 In summary, PR produced traffic behavior that is consistent with field observations,
354 hence the preferred alternative in this research to define yielding logic for roundabouts.
355 Therefore, all remaining analyses used PR.

356



a. Distribution of Accepted Gaps (Conflict Areas vs. Priority Rules)



b. Distribution of Maximum Rejected Gaps (Conflict Areas vs. Priority Rules)

FIGURE 3 Comparison between conflict area and priority rule.

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368

Sensitivity Analyses

The following subsections are dedicated to sensitivity analyses of settings of different VISSIM elements to investigate their impact on the roundabout model’s capacity (i.e., t_c and t_f). The results of the sensitivity analysis are expected to provide quantitative reference for calibrating the roundabout simulation model. Specifically, the VISSIM elements that are considered in this section include:

- PR: minimum gap;
- RSA: speed distribution and deceleration rate;
- Wiedemann 74 Model (W74M): safety distance factors: additive and multiplicative.

369 **TABLE 2 Settings of Parameters and Results of the Sensitivity Analyses**

Priority Rules				Parameter		Result	
				Minimum Gap (s)		t_c (s)*	t_f (s)*
	Default Value			4.3			
Value in Sensitivity Analysis	Experiment ID	1	3.0	3.74 (0.13)	2.81 (0.45)		
		2	3.5	4.24 (0.16)	2.82 (0.44)		
		3	4.0	4.68 (0.18)	2.82 (0.44)		
		4	4.3	4.90 (0.19)	2.84 (0.43)		
		5	4.5	5.04 (0.22)	2.84 (0.45)		
		6	5.0	5.39 (0.29)	2.84 (0.44)		
		7	5.5	5.72 (0.43)	2.86 (0.49)		

Reduced Speed Areas				Parameter		Result	
				Speed Distribution (mph)	Deceleration Rate (ft/s ²)	t_c (s)	t_f (s)
	Default Value			$V_{50th}^* = 13.2; V_{85th}^* = 18.0$	4.19		
Value in Sensitivity Analysis	Experiment ID	11	$V_{50th} = 13.2; V_{85th} = 18.0$	1.19	4.84 (0.26)	2.84 (0.45)	
		12	$V_{50th} = 13.2; V_{85th} = 18.0$	3.19	4.88 (0.28)	2.84 (0.44)	
		13	$V_{50th} = 13.2; V_{85th} = 18.0$	4.19	4.90 (0.19)	2.84 (0.43)	
		14	$V_{50th} = 13.2; V_{85th} = 18.0$	5.19	4.92 (0.29)	2.84 (0.44)	
		15	$V_{50th} = 13.2; V_{85th} = 18.0$	7.19	4.91 (0.31)	2.83 (0.43)	
		16	$V_{50th} = 8.2; V_{85th} = 13.0$	4.19	4.75 (0.15)	3.34 (0.58)	
		17	$V_{50th} = 11.2; V_{85th} = 16.0$	4.19	4.80 (0.18)	3.02 (0.50)	
		18	$V_{50th} = 13.2; V_{85th} = 18.0$	4.19	4.90 (0.19)	2.84 (0.43)	
		19	$V_{50th} = 15.2; V_{85th} = 20.0$	4.19	4.95 (0.21)	2.77 (0.45)	
		20	$V_{50th} = 18.2; V_{85th} = 23.0$	4.19	5.09 (0.26)	2.72 (0.47)	

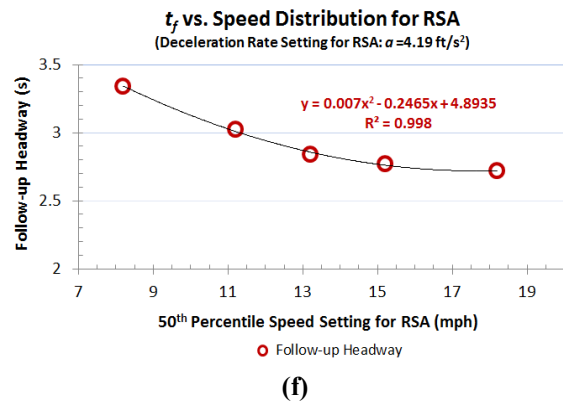
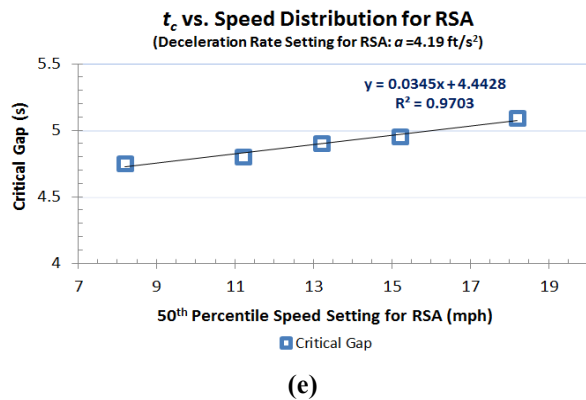
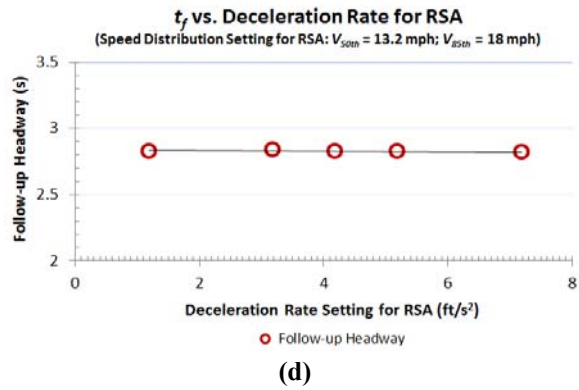
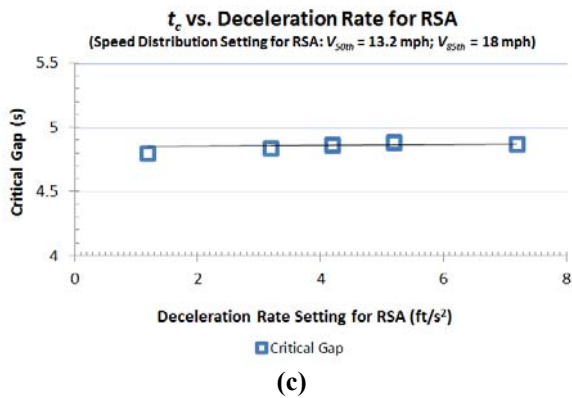
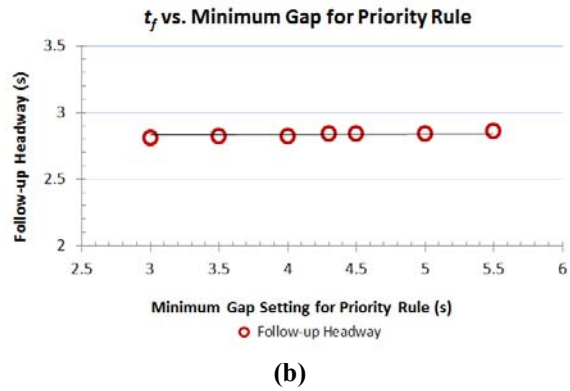
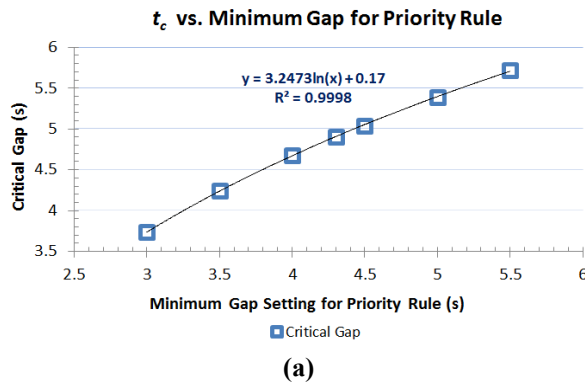
Wiedemann 74 model				Parameter		Result	
				Additive Part of Safety Distance	Multiplicative Part of Safety Distance	t_c (s)	t_f (s)
	Default Value			2.0	3.0		
Value in Sensitivity Analysis	Experiment ID	21	2.0	1.5	4.89 (0.21)	2.60 (0.40)	
		22	2.0	2.5	4.87 (0.20)	2.76 (0.43)	
		23	2.0	3.0	4.90 (0.19)	2.84 (0.43)	
		24	2.0	3.5	4.88 (0.19)	2.92 (0.45)	
		25	2.0	4.5	4.90 (0.19)	3.08 (0.47)	
		26	1.0	3.0	4.84 (0.20)	2.53 (0.43)	
		27	1.5	3.0	4.85 (0.20)	2.68 (0.42)	
		28	2.0	3.0	4.90 (0.19)	2.84 (0.43)	
		29	2.5	3.0	4.93 (0.21)	3.01 (0.44)	
		30	3.0	3.0	4.93 (0.20)	3.17 (0.44)	

* t_c denotes critical gap; t_f denotes follow-up headway; () denotes standard deviation; V_{50th} denotes 50th percentile speed; V_{85th} denotes 85th percentile speed.

370
 371 The main idea of the sensitivity analysis was to test how sensitive the changes in critical
 372 gap (t_c) and follow-up headway (t_f) were when changing parameter values of a subject VISSIM
 373 element. Since the left lane of the NB entrance was selected as the study lane in this research,

374 changes in parameter settings only applied to the NB left lane in the sensitivity analyses, except
 375 changes in some global settings like the W74M. In each analysis, only the parameter values of
 376 the subject VISSIM element were changed. The parameter values of other elements remained at
 377 defaults. For RSA and PR, the default parameter values were the field-observed values. For
 378 W74M, the default parameter values were values that are recommended by VISSIM (20). Each
 379 analysis included multiple experiments. Table 2 summarizes the settings of VISSIM parameters
 380 and the results of the sensitivity analyses.

381



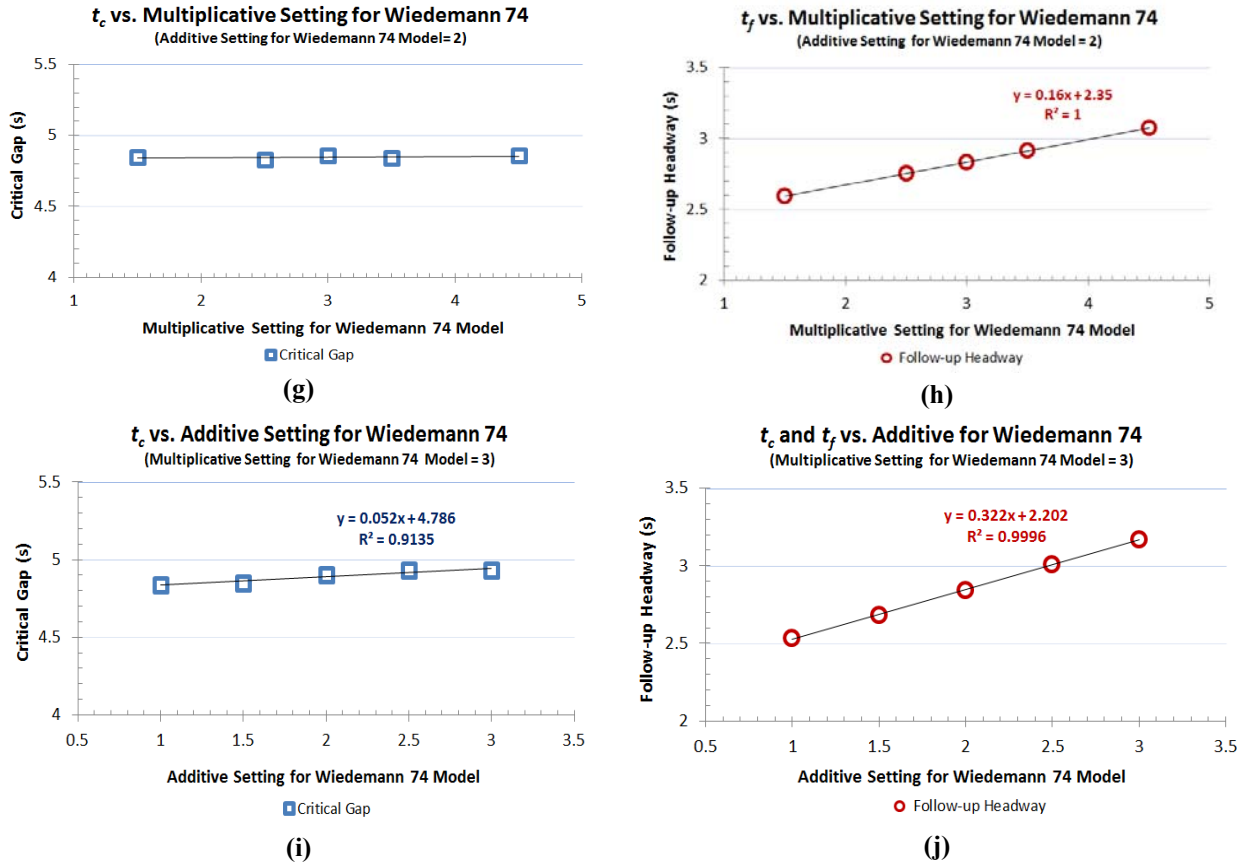


FIGURE 4 Numerical illustrations of sensitivity analysis results.

382

383

384 *Results of Priority Rule*

385 The sensitivity analysis for PR was designed to quantify the impact of the PR’s minimum gap
 386 setting on the roundabout capacity. The setting of minimum gap in the PR is a close reflection of
 387 the critical gap. Before running the analysis, it was expected that linear relationship lies between
 388 the minimum gap and the critical gap. For instance, increasing the minimum gap by 0.1 second
 389 would also increase the critical gap by 0.1 second. The sensitivity analysis aimed at verifying
 390 this expectation.

391 Seven different minimum gaps, ranging from 3.0 sec to 5.5 sec, were tested in the
 392 analysis. Figures 4.a and 4.b illustrate the results of the minimum gap’s impact on critical gap
 393 and follow-up headway, respectively. As expected, increasing the minimum gap significantly
 394 increased the critical gap according to Figure 4.a. And, all critical gaps were observed being
 395 greater than the minimum gaps. For instance, when inputting the minimum gap as 4.3 sec, a
 396 critical gap of 4.9 sec was observed. However, the difference between the input minimum gap
 397 and the resulted critical gap was not constant through all the tested minimum gaps, which
 398 suggests that the minimum gap input and the resulting critical gap are not linearly correlated.
 399 Regression analysis also identified the best-fit numerical relationship between the minimum gap
 400 and the resulted critical gap (under default settings summarized in Table 2) to be logarithmic
 401 rather than linear as initially expected. Numerically, the relationship is represented by the
 402 following equation:

$$\begin{aligned}
 403 \quad t_c &= 3.2473 \ln(g_{\min}) + 0.17 \\
 R^2 &= 0.9998
 \end{aligned}
 \tag{3}$$

404 Where, t_c is critical gap (sec), and g_{\min} is minimum gap setting for the PR (sec).

405 Increasing the minimum gap did not change the follow-up headway according to Figure
 406 4.b. The regression line was quite flat, indicating that the minimum gap did not have a significant
 407 impact on the follow-up headway as expected.

408 *Results of Reduced Speed Area*

409 Although the location, speed distribution, and deceleration rate of RSA have been determined
 410 using field observation in the previous section, sensitivity analyses of RSA settings were still
 411 required to investigate how these settings would quantitatively impact the roundabout capacity.
 412 The analysis was comprised of two sets of sensitivity analyses. The first set was designed to test
 413 the impact of RSA's deceleration rate on the roundabout capacity. In the analysis, the RSA's
 414 speed distribution remained the default value (i.e., 50th percentile speed = 13.2 mph and 85th
 415 percentile speed = 18.0 mph). Five different deceleration rates, ranging from 1.19 ft/s² to
 416 7.19 ft/s², were tested in the analysis. Figures 4.c and 4.d illustrate the results of the deceleration
 417 rate's impact on critical gap and follow-up headway, respectively. According to Figure 4.c,
 418 increasing the deceleration rate did not change the critical gap significantly. In other words, the
 419 deceleration rate only controlled the starting distance of drivers' deceleration maneuver. It did
 420 not impact drivers' critical gap. Similar result was found for follow-up headway according to
 421 Figure 4.d. The regression curve was flat, indicating that the deceleration rate did not have
 422 significant impact on drivers' follow-up headway, either.

423 The second set of analysis was designed to test the impact of RSA's speed distribution
 424 setting on the roundabout capacity. In the analysis, the RSA deceleration rate remained at the
 425 default value (i.e., deceleration rate = 4.19 ft/s²). Five different speed distributions with 50th
 426 percentile speed ranging from 8.2 mph to 18.2 mph were tested in the analysis. Figures 4.e and
 427 4.f illustrate the analysis results. Figure 4.e showed that increasing the 50th and 85th percentile
 428 speeds in the speed distribution significantly increased the critical gap. Through regression
 429 analysis, a linear relationship between the 50th percentile speed and the critical gap (under default
 430 settings summarized in Table 2) was identified, and was represented by the following equation:

$$\begin{aligned}
 432 \quad t_c &= 0.0345v_{50th} + 4.4428 \\
 R^2 &= 0.9703
 \end{aligned}
 \tag{4}$$

433 Where, t_c is critical gap (sec), and v_{50th} is 50th percentile speed in the RSA's speed distribution
 434 setting (mph).

435 In regards to follow-up headway, increasing the 50th and 85th percentile speeds in the
 436 speed distribution for RSA was found to significantly reduce the follow-up headway as shown in
 437 Figure 4.f. Rather than in a linear form, the increment in the 50th percentile speed tended to
 438 reduce the follow-up headway in a polynomial form. The magnitude of reduction was high in the
 439 lower speed range (8.2-11.2 mph) and was low in the higher speed range (15.2-18.2 mph).
 440 Through regression analysis, the relationship between the 50th percentile speed and the follow-up
 441 headway (under default settings summarized in Table 2) was represented by the following
 442 equation:

$$\begin{aligned}
 443 \quad t_f &= 0.007v_{50th}^2 - 0.2465v_{50th} + 4.8935 \\
 R^2 &= 0.998
 \end{aligned}
 \tag{5}$$

444 Where, t_f is follow-up headway (sec), and v_{50th} is 50th percentile speed in the RSA's speed
 445 distribution setting (mph).

446

447 *Results of the Wiedemann 74 Model*

448 In VISSIM, the W74M includes two important adjustable parameters. Namely, the additive and
 449 multiplicative parts of the safety distance. Therefore, the analysis of the W74M was comprised
 450 of two sets of sensitivity analyses. The first set was designed to test the impact of the
 451 multiplicative setting on the roundabout capacity. In the analysis, the additive setting remained
 452 the default value (i.e., additive = 2.0). Five different multiplicative settings, ranging from 1.5 to
 453 4.5, were tested in the analysis. Figures 4.g and 4.h illustrate the results of the multiplicative
 454 setting's impact on critical gap and follow-up headway, respectively. According to Figure 4.g,
 455 increasing the multiplicative setting did not change the critical gap significantly. In other words,
 456 the multiplicative setting had no impact on driver's critical gap. However, it did have significant
 457 impact on the follow-up headway according to Figure 4.h. The follow-up headway increased
 458 linearly with the multiplicative setting. Generally, for one unit increment of the multiplicative
 459 setting, the follow-up headway increased by about 0.16 sec. Regression analysis summarized the
 460 relationship between the multiplicative setting and the follow-up headway (under default settings
 461 summarized in Table 2) using the following equation:

$$\begin{aligned}
 462 \quad t_f &= 0.16m + 2.35 \\
 R^2 &= 1
 \end{aligned}
 \tag{6}$$

463 Where, t_f is follow-up headway (sec), and m is the multiplicative part of the safety distance for
 464 the W74M.

465 The second set of analysis was designed to test the impact of the additive setting on the
 466 roundabout capacity. In the analysis, the multiplicative setting remained the default value (i.e.,
 467 multiplicative = 3.0). Five different additive settings, ranging from 1.0 to 3.0, were tested in the
 468 analysis. Figures 4.i and 4.j illustrate the analysis results. It was identified from Figure 4.i that
 469 increasing the additive setting slightly increased the critical gap. Through regression analysis, a
 470 linear relationship between the 50th percentile speed and the critical gap (under default settings
 471 summarized in Table 2) was identified, and was represented by the following equation:

$$\begin{aligned}
 472 \quad t_c &= 0.052a + 4.786 \\
 R^2 &= 0.9135
 \end{aligned}
 \tag{7}$$

473 Where, t_c is critical gap (sec), and a is the additive part of the safety distance for the W74M.

474 Increasing the additive setting was found to increase the follow-up headway in a linear
 475 form according to Figure 4.j. The magnitude of increment in follow-up headway is 0.32 sec per
 476 unit increment in the additive setting. Through regression analysis, the relationship between the
 477 50th percentile speed and the follow-up headway (under default settings summarized in Table (2)
 478 was represented by the following equation:

$$t_f = 0.322a + 2.202$$
$$R^2 = 0.9996$$
(8)

479

480 Where, t_f is follow-up headway (sec), and a is additive part of the safety distance for the W74M.

481

482 CALIBRATION RECOMMENDATIONS

483 The following recommendations have been made for calibrating VISSIM roundabout models
484 based on the sensitivity analysis results:

485 • The entrance RSA is recommended to be placed at approximately 8 feet from the yield
486 line, and the length of the RSA is 17 feet. The speed distribution setting is recommended
487 to conform to the speed cumulative curve shown in Figure 2.c. Since the RSA's
488 deceleration rate setting does not impact the roundabout capacity, the deceleration rate
489 can vary to adapt the actual approach speed of a specific roundabout. For example, higher
490 speed approach can use a higher deceleration rate. The chosen deceleration rate is
491 recommended to fall within the field-observed distribution shown in Figure 2.b.

492 • PRs rather than CAs are recommended to model the yielding logics to achieve consistent
493 and repeatable gap acceptance behavior.

494 • Speed distribution of RSA and the additive setting for W74M are *not* recommended to be
495 used in calibrating the roundabout model, as they impact both the critical gap and follow-
496 up headway simultaneously. Using these two parameters will make the calibration hard to
497 control. Therefore, the RSA's speed distribution and the additive setting for W74M are
498 recommended to remain default values (i.e., Figure 2.c for speed distribution, and 2.0 for
499 additive setting) during the calibration.

500 • Minimum gap for PR impacts the critical gap only, and hence is recommended to be used
501 in calibrating the critical gap. The critical gap can be calculated by inputting the
502 minimum gap into Equation (3).

503 • Multiplicative setting for W74M impacts the follow-up headway only, and hence is
504 recommended to be used in calibrating the follow-up headway. The follow-up headway
505 can be calculated using Equation (6) and a multiplicative input.

506

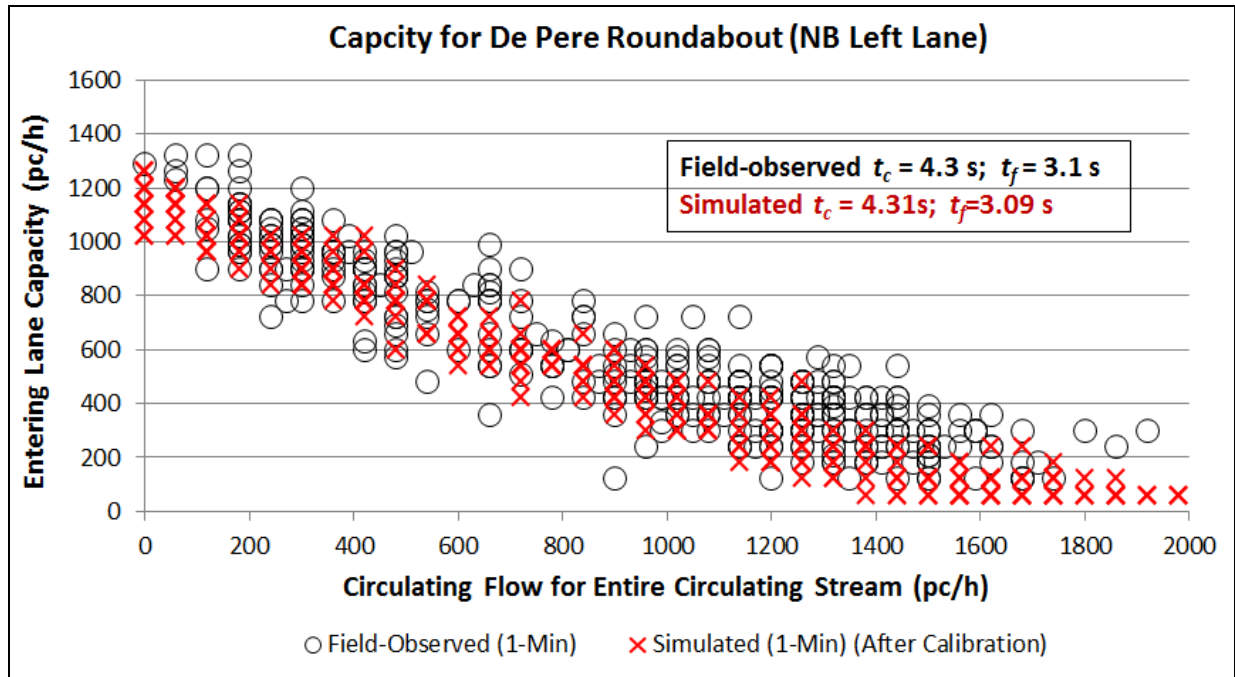
507 VALIDATION OF THE CALBRATION RECOMMENDATIONS

508 Using the study site as a case study, the effectiveness of the calibration recommendations
509 developed in the previous section is validated in this section.

510 According to Table 1, the study entrance lane (i.e., NB left lane of De Pere roundabout)
511 had a field-observed critical gap of 4.3 sec and follow-up headway of 3.1 sec. In the validation
512 process, the study lane's critical gap and follow-up headway were attempted to be calibrated
513 following the calibration recommendations. Specifically, the minimum gap of 3.6 sec and the
514 multiplicative value of 4.7 were used based on Equations (3) and (6) in order to obtain the
515 4.3 sec critical gap and the 3.1 sec follow-up headway. All other VISSIM parameters remained
516 default values as summarized in Table 2. Validation results obtained from 150 simulation runs
517 showed that the resultant critical gap and follow-up headway after calibration were 4.31 sec and
518 3.09 sec, respectively. Both numbers were almost identical to the calibration goal. Figure 5
519 further illustrates the validation of the calibrated simulation model by comparing the simulated

520 capacity cloud (i.e., plots of entering flow vs. circulating flow) with the field-observed ground-
 521 truth capacity cloud.

522



523

524

FIGURE 5 Validation of the roundabout simulation model.

525 According to Figure 5, the capacity cloud obtained from the calibrated simulation model
 526 matches very well with the field-observed capacity cloud, which indicates that the VISSIM
 527 roundabout model was successfully calibrated. By calculating the root-mean-square-error, the
 528 difference between the average VISSIM data and field data was about 116 pc/hr/ln. At very high
 529 circulating flows above 1400 pc/hr, field data, although sparse, showed slightly more capacity
 530 than the simulated data. Based on these validation results, the proposed calibration
 531 recommendations for VISSIM roundabout models demonstrated their applicability in calibrating
 532 VISSIM roundabout models. .

533

534 CONCLUSIONS

535 Based on the findings presented in the previous sections, the following conclusions were
 536 reached:

- 537 • *Field estimates of RSA parameters:* location and length for RSAs have been determined
 538 through analyzing the four stages of drivers' deceleration maneuver based on field speed
 539 data. The estimates for RSA deceleration rate and speed distribution have also been
 540 determined as shown by Figure 2.b and Figure 2.c.
- 541 • *CA vs. PR:* using CAs may result in circulating vehicles yielding to entering vehicles,
 542 while using PRs may not. The finding concurs with Wei et al.'s finding (4). However,
 543 when the links and connectors are modeled properly, the yielding behavior of the
 544 circulating traffic can be eliminated. Compared with PRs, CAs have wider and
 545 inconsistent distribution of accepted gaps and largest rejected gaps, which indicates using

546 PR can result in more uniform and repeatable behavior in accepting and rejecting gaps.
 547 Therefore, PRs are recommended for modeling roundabouts.

- 548 • *Sensitivity analyses of VISSIM parameters*
 - 549 ○ *Minimum gap for PR*: critical gap increases as minimum gap increases, the numerical
 550 relationship between them is logarithmic; and, follow-up headway is not impacted by
 551 minimum gap.
 - 552 ○ *Deceleration rate for RSA*: both critical gap and follow-up headway are not impacted
 553 by RSA's deceleration rate; and, deceleration rate only controls the starting distance
 554 of driver's deceleration maneuver in VISSIM.
 - 555 ○ *Speed distribution for RSA*: critical gap increases as RSA's 50th and 85th percentile
 556 speeds increase; the numerical relationship between 50th percentile speed and critical
 557 gap is linear; follow-up headway decreases as RSA's 50th and 85th percentile speeds
 558 increase; and, the numerical relationship between 50th percentile speed and follow-up
 559 headway is polynomial.
 - 560 ○ *Multiplicative setting for the W74M*: critical gap is not impacted by the multiplicative
 561 setting; follow-up headway increases as multiplicative increases; and, the numerical
 562 relationship between multiplicative and follow-up headway is linear;
 - 563 ○ *Additive setting for the W74M*: critical gap slightly increases as additive increases; the
 564 numerical relationship between additive and critical gap is linear; follow-up headway
 565 increases as additive increases; and, the numerical relationship between additive and
 566 follow-up headway is linear;

567 In summary, the paper tries to develop simple and numerical calibration
 568 recommendations based on the comprehensive discussion of the calibration process. Despite the
 569 complex process of developing such recommendations, the final product is simple and it
 570 provides formulated solutions to researchers and practitioners to simplify their calibration of
 571 VISSIM roundabout models. The limitation is that the calibration recommendations were
 572 validated based on the data from only one roundabout. Future research will focus on
 573 investigating the transferability of the proposed calibration recommendations via exploring more
 574 study sites in the validation process.

575

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 580 for estimating critical gap.

581

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